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Aidrian V. Coward
Philip Hall

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ON THE NONLINEAR INTERFACIAL INSTABILITY OF ROTATING CORE-ANNULAR FLOW

Aidrian V. Coward and Philip Hall[1]
Department of Mathematics
University of Manchester
Manchester M13 9PL
United Kingdom

ABSTRACT

The interfacial stability of rotating core-annular flows is investigated. The linear and nonlinear effects are considered for the case when the annular region is very thin. Both asymptotic and numerical methods are used to solve the flow in the core and film regions which are coupled by a difference in viscosity and density. The long-time behaviour of the fluid-fluid interface is determined by deriving its nonlinear evolution in the form of a modified Kuramoto-Sivashinsky equation. We obtain a generalization of this equation to three dimensions. The flows considered are applicable to a wide array of physical problems where liquid films are used to lubricate higher or lower viscosity core fluids, for which a concentric arrangement is desired. Linearized solutions show that the effects of density and viscosity stratification are crucial to the stability of the interface. Rotation generally destabilizes non-axisymmetric disturbances to the interface, whereas the centripetal forces tend to stabilize flows in which the film contains the heavier fluid. Nonlinear affects allow finite amplitude helically travelling waves to exist when the fluids have different viscosities.

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1 Introduction

In 1967 Yih [20] studied the linearized long-wave stability problem for the parallel flow of two superposed fluids trapped between infinite flat plates. He showed analytically that Poiseuille and Plane Couette flows of viscously stratified fluids may be unstable to interfacial disturbances. Energy is transferred from the moving boundary or the applied pressure gradient to the wavy motion of the interface. Yih also found that the volume ratio of the two liquids is a crucial factor in determining the flow stability. Moreover, when the less viscous liquid layer is thin, the flow is found to be stable to disturbances with streamwise wavelengths much longer than the plate separation.

Since this novel work there has been much interest in viscously stratified flows and their wide-spread practical applications. An example of this is the study of water waves generated by wind as considered by Blennerhassett [2]. He used Yih's long-wave approach to determine the growth of small amplitude surface waves for fully developed laminar flow of air over water, and then extended this to include the weakly nonlinear effects and determine the wave amplitude equation. The numerical results suggest that the nonlinear effects reduce the growth rate and lead to equilibrium amplitude waves.

Another practical application of this theory arises from the difficulties encountered in transporting highly viscous liquids, such as crude oils. These difficulties can be overcome by heating, or diluting the fluid, such methods, however are often considered expensive and impractical, and a desirable alternative is to lubricate the flow using another immiscible fluid. Joseph, Renardy and Renardy [12] investigated the validity of the 'viscous dissipation principle' and concluded that a less viscous lubricating film has a tendency to encapsulate a heavy liquid core.

Flows of concentric, immiscible fluids inside pipes are known as core-annular flows, hereinafter referred to as CAF. The most simple arrangement is when one liquid occupies the central core region and is surrounded by another filling the annulus. Our interest lies with such an arrangement inside a cylindrical pipe, known as a two phase CAF. When stable, the two fluids form concentric regions aligned parallel to the pipe wall. In practice such an ideal arrangement is not realised purely by chance and a question of some importance is to determine the circumstances under which this geometry is altered. For bicomponent flows of oil and water, (or some similar aqueous solution), in cylindrical CAFs practical observations include the formation of slugs of oil and water, wavy core flows and emulsions of both fluids.

The linear stability of fluid-fluid interfaces has been studied comprehensively by a number of authors. Hickox [7] considered cylindrical CAF in which the less viscous fluid occupied the core region and concluded that the arrangement was always unstable to long waves. Joseph, Renardy and Renardy [12] studied the inverse problem, that is, a CAF with a more viscous core, they too confirmed Yih's findings (for a planar geometry), namely that the flow is stable to axisymmetric long wave disturbances. The effects of surface tension and gravity were neglected in this work, but for planar geometries the destabilizing influence of density stratification can be overcome by the stabilizing effects of surface tension and viscosity differences, provided that the depth of the less viscous fluid is made small enough, see Renardy [19]. Hooper and Boyd [8] indicated that it is short waves which are stabilized the most.

The effect of surface tension on CAF is more complicated. Preziosi, Chen and Joseph [18] undertook a detailed linear stability analysis for an oil core flow lubricated by water in a two phase CAF, their findings are largely in agreement with the experimental work of Charles, Govier and Hodgson [3]. They conclude that surface tension stabilizes short waves and destabilizes long waves. The generalised capillary instability of long waves below the lower branch of the neutral curve is thought to lead to oil slugs and bubbles forming in the lubricating water layer. Preziosi et al. also investigated the influence of density stratification in the absence of gravity, they show that it has opposite effects at the upper and lower branches of the neutral curve. Increasing the density of the fluid in the annulus relative to that of the core destabilizes disturbances near the upper branch and to a lesser extent, stabilizes disturbances near the lower branch.

Hu and Joseph [9] studied three arrangements of CAF: (i) An oil core with water annulus; (ii) A water core and oil annulus; and (iii) a three phase CAF with a water layer trapped between oil regions. They show that for the second case the flow is always unstable, but only weakly so when the oil film is thin. They also note that in all cases, when the Reynolds number is small it is the capillary instability which is most dominant.

In much of the work mentioned above the influence of gravity is neglected. Both Chen, Bai and Joseph [4] and Georgiou, Maldarelli, Papageorgiou and Rumschitzki [6] considered its effect when the pipe is aligned vertically, so as not to violate the axisymmetry of the problem. The former authors found that gravity had a destabilizing effect at all Reynolds numbers with either a light or heavy lubricant. In particular they conclude that for slow flow, a heavy lubricant should be used when the pressure gradient acts in the same sense as the gravitational force, a lighter one when the flow acts upwards. Georgiou et al. [6] made a full analytical analysis of the linear stability of

CAF in which the depth of the outer layer is asymptotically small. They make several important conclusions, namely that the inertia of the film does not affect the stability at leading order and that for moderate surface tension it is the viscosity stratification which is dominant. There work was found to be in excellent agreement with the above authors when the depth of the film becomes larger.

Miesen, Beijnon, Duijvestijn, Oliemans and Verheggen [14] found excellent qualitative agreement between the theoretical and experimental study of interfacial waves in CAF. They explain that the absence of good quantitative results, especially when the wave amplitude was observed to be large, is most likely due to the ommission of nonlinear effects.

In all this work so far we have discussed the case of a fixed pipe, and it has been found that the axisymmetric modes are the most dangerous. The effects of rotation on the stability of CAF are of practical importance, an example of this is in the oil industry where fluids are pumped through boreholes containing rotating drills. The study of a *single* rotating fluid has of course been undertaken by many authors, it is widely accepted that the most unstable disturbances are the axisymmetric modes. Hu and Joseph [10] performed a linear stability analysis for a two phase rotating CAF. They conclude that rotation stabilized the axisymmetric ($n = 0$) modes and destabilized the non-axisymmetric ($n = 1$) modes which were found to be the most dangerous. When the more dense fluid is in the annulus, there exists a critical rotational speed, above which the centripetal acceleration stabilizes the flow. With the lighter fluid in the annulus the flow is always unstable.

In this work we study the weakly nonlinear interfacial stability of a rotating two phase CAF, for both axisymmetric and non-axisymmetric disturbances to the interface between the core and annular regions. The procedure used here closely follows that of Papageorgiòu et al. [15] who studied the fixed pipe axisymmetric problem. In particular we consider the case when the annular region is very thin compared with the pipe radius, so that an asymptotic expansion can be made in terms of a single small parameter. We aim to analyse the effect of rotation on the perturbation to the concentric basic flow when the viscosities of the core and film layers are different. We also allow for a small difference in the densities of the two fluids, but the effects of gravity are neglected. Frenkel, Babchin, Livich, Shlang and Sivashinsky [5] considered the simplified axisymmetric case when the viscosities of the film and core were equal so that the dynamics of the core did not affect the nonlinear stability problem. They obtained the

Kuramoto-Sivashinsky (KS) equation which governs the evolution of the interface, it can be written in normalized form as:

$$\eta_t + \eta\eta_z + \eta_{zz} + \lambda\eta_{zzzz} = 0,$$

where t denotes time and z the axial distance along the pipe.
The KS equation arises in many physical situations such as flame propagation and thin film flow down an inclined plane. Extensive numerical studies on the route to chaos taken by solutions of the KS equation on a spatially periodic domain have been undertaken by Papageorgiou and Smyrlis [16] and [17]. They observe many interesting characteristics shown by this equation, such as, steady modal attractors, periodic attractors and chaotic oscillations. The accuracy of their numerical experiments have identified windows, (ranges in the parameter λ), where the time periodic solutions undergo a complete sequence of up to thirteen period doubling bifurcations. This has enabled the authors to compute Feigenbaum's universal constant to a high degree of accuracy.

Papageorgiou, Maldarelli and Rumschitzki [15] included the dynamics of the core which are coupled to the film flow by the viscosity difference. They found that a necessary constraint for the inclusion of capillary effects was that $\epsilon J \sim R_e$ where J is a non-dimensional surface tension parameter, R_e the Reynolds number based on the core speed, and ϵ the ratio of the film depth to the radius of the core. They satisfied this by considering two physical regimes:

A: slow core flow with moderate surface tension,

B: moderate core flow with large surface tension.

The resulting evolution equation is a modified form of the KS equation, it includes an additional term which represents the effect of the core dynamics. For case (A) this term has a purely dispersive effect because the slow flow gives rise to a Stokes flow approximation in which the inertial effects of the core are neglected. When either the more or less viscous fluid occupies the film, they find that this dissipative term tends to organise the otherwise chaotic motion of the interface into travelling waves. For the case of much larger surface tension (B), they find that viscous stratification also has a linearly stabilizing influence when the core is the more viscous of the two fluids, for a less viscous core it is linearly destabilizing. Never the less the imaginary contribution still has this same ability to produce travelling waves.

When rotation is introduced the method used by Papageorgiou et al. can be adapted to find the solution to the non-axisymmetric problem. We solve the core problem numerically as well as analytically for two specific cases, namely, a slow Stokes flow without rotation and secondly when rotation is asymptotically large. Having taken a thin film limit, a lubrication approximation is valid in the film layer and the solution in that region follows. The method by which the core dynamics are coupled with the motion of the film is through the equations of tangential stress, namely the radial derivatives of the axial and azimuthal flow.

The capillary effects are retained if the speed of the core satisfies a similar constraint to the one discussed above. As we shall see later however, this is altered somewhat by rotation, the cenripetal pinch effect on the interface requires an even slower core flow to realise capillary effects. In other words, for a given Reynolds number, (that is, for a given axial speed), increasing rotation reduces the effect of surface tension.

A solution is found for the disturbed flow in both the core and film regions and the evolution equation is derived. For the axisymmetric case, associated with rotating cylindrical CAF it is not surprisingly found that the nonlinear evolution equation which includes capillary and viscous effects is also a modified form of the Kuramoto-Sivashinsky equation. The rotation alters the additional linear viscous coupling term found by Papageorgiou et al. and its effect is discussed later. The non-axisymmetric problem extends the evolution equation to a novel three dimensional form of the modified KS equation.

As rotation increases an interfacial boundary layer is formed inside the core layer. The analysis used here requires the thickness of the film to remain smaller than the depth of this layer, as a result we have an upper bound for the rotation of the cylinder.

The layout of this work is as follows: in §2 we derive the non-dimensional basic flow for a rotating cylinder containing two concentric fluids; in §3 we take the limit of small film thickness and make a lubrication type approximation in the film. The core problem is then solved both numerically and analytically. The solution in the film is found and the evolution equation derived. In §4 we present the methods used to solve the linear and nonlinear problems and discuss the numerical results. Finally, in section §5 we draw some conclusions.

2 Geometry and Basic Flow

Two incompressible, viscous fluids are contained in an infinitely long circular cylinder. The fluids have constant, but different densities and kinematic viscosities, and they are immiscible. We use cylindrical polar coordinates (r^*, θ^*, z^*) to denote the radial, azimuthal and axial directions respectively, so that the cylinder has radius R_2 and its axis lies along $r^* = 0$. The interface between the two fluids is at a constant radial distance R_1, so that they occupy two distinct, concentric regions:

CORE REGION : Fluid of density ρ_c and kinematic viscosity μ_c occupies $0 \leq r^* \leq R_1$,
FILM REGION : Fluid of density ρ_f and kinematic viscosity μ_f occupies $R_1 \leq r^* \leq R_2$.

The cylinder rotates with angular velocity ω and there is a prescribed, constant axial pressure gradient $-F$, (such that $F > 0$) but the effect of gravity is neglected. We seek an exact solution of the Navier Stokes equations of the form $\overline{\mathbf{U}}^*_{c,f}(r^*) = (0, \overline{V}^*, \overline{W}^*)_{c,f}$ and pressure $\overline{P}^*_{c,f}(r^*)$, where the subscripts c, f denote a core and film quantity respectively. There is a discontinuity in the film and core pressure fields due to the surface tension σ between the two fluids, (see Lamb [13]), hence:

$$\overline{P}^*_c(r^* = R_1) - \overline{P}^*_f(r^* = R_1) = \frac{\sigma}{R_1}.$$

The basic flow is:

$$\begin{aligned}
\overline{V}^*_c(r^*) &= \omega r^*, \\
\overline{V}^*_f(r^*) &= \omega r^*, \\
\overline{W}^*_c(r^*) &= \frac{F}{4\mu_c}\left(R_1^2 - r^{*^2}\right) + \frac{F}{4\mu_f}\left(R_2^2 - R_1^2\right), \\
\overline{W}^*_f(r^*) &= \frac{F}{4\mu_f}\left(R_2^2 - r^{*^2}\right).
\end{aligned}$$

We non-dimensionalise the above quantities using a velocity scale W_0, the primary flow evaluated at the axis of the cylinder,

$$W_0 = \overline{W}^*_c(r^* = 0) = \frac{F R_1^2}{4\mu_c} + \frac{F}{4\mu_f}\left(R_2^2 - R_1^2\right).$$

The interface radius R_1 is used as a radial and axial length scale, whilst $\frac{R_1}{W_0}$ and $\rho_c W_0^2$ are the appropriate scales for the time and pressure respectively. We also define

$$R_e = \frac{W_0 R_1 \rho_c}{\mu_c}, \qquad \Omega = \frac{\omega R_1 R_e}{W_0},$$

$$m = \frac{\mu_f}{\mu_c}, \qquad \rho = \frac{\rho_f}{\rho_c}, \qquad a = \frac{R_2}{R_1}.$$

Here R_e is the Reynolds number and Ω is a rotation parameter. Using non-dimensional polar coordinates defined by $(r^*, \theta^*, z^*) = (R_1 r, \theta, R_1 z)$, the primary flow may be written in the form

$$\overline{V}_c(r) = \frac{\Omega r}{R_e}, \tag{2.1a}$$

$$\overline{V}_f(r) = \frac{\Omega r}{R_e}, \tag{2.1b}$$

$$\overline{W}_c(r) = 1 - \frac{m r^2}{a^2 + m - 1}, \tag{2.1c}$$

$$\overline{W}_f(r) = \frac{a^2 - r^2}{a^2 + m - 1}. \tag{2.1d}$$

Consider now, a more general fluid velocity $(U, V, W)_{c,f}$ and pressure $P_{c,f}$ which are also functions of the azimuthal coordinate θ. The non-dimensional governing equations in the core are:

$$U_t + UU_r + \frac{VU_\theta}{r} + WU_z - \frac{V^2}{r} = -P_r + \frac{1}{R_e}\left(\nabla^2 U - \frac{U}{r^2} - \frac{2V_\theta}{r^2}\right), \tag{2.2a}$$

$$V_t + UV_r + \frac{VV_\theta}{r} + WV_z + \frac{UV}{r} = -\frac{P_\theta}{r} + \frac{1}{R_e}\left(\nabla^2 V - \frac{V}{r^2} + \frac{2U_\theta}{r^2}\right), \tag{2.2b}$$

$$W_t + UW_r + \frac{VW_\theta}{r} + WW_z = -P_z + \frac{1}{R_e}\left(\nabla^2 W\right), \tag{2.2c}$$

and in the film:

$$U_t + UU_r + \frac{VU_\theta}{r} + WU_z - \frac{V^2}{r} = -\frac{P_r}{\rho} + \frac{m}{\rho R_e}\left(\nabla^2 U - \frac{U}{r^2} - \frac{2V_\theta}{r^2}\right), \tag{2.3a}$$

$$V_t + UV_r + \frac{VV_\theta}{r} + WV_z + \frac{UV}{r} = -\frac{P_\theta}{\rho r} + \frac{m}{\rho R_e}\left(\nabla^2 V - \frac{V}{r^2} + \frac{2U_\theta}{r^2}\right), \tag{2.3b}$$

$$W_t + UW_r + \frac{VW_\theta}{r} + WW_z = -\frac{P_z}{\rho} + \frac{m}{\rho R_e}\left(\nabla^2 W\right),, \tag{2.3c}$$

where the Laplacian operator $\nabla^2(.) \equiv \left((.)_{rr} + \frac{(.)_r}{r} + (.)_{zz} + \frac{(.)_{\theta\theta}}{r^2}\right)$ and the subscripts denote partial differentiation.

The equation of continuity must be satisfied by both the core and film fluids and takes the form

$$(Ur)_r + V_\theta + rW_z = 0. \tag{2.4}$$

The film flow must satisfy no slip at the cylinder wall, $(r = a)$, and in the core the velocity and pressure must remain finite at the axis. We now allow the interface to be expressed more generally by $r = S(\theta, z, t)$. Here we must satisfy continuity of velocity so that,

$$U_c = U_f, \qquad V_c = V_f, \qquad W_c = W_f. \tag{2.5}$$

For the continuity of tangential and normal stress we first resolve into directions parallel and perpendicular to the interface, we use the jump notation $[.] \equiv (.)_c - (.)_f$, evaluated at $r = S$ to obtain:

$$\begin{aligned} [S_z F_r + F_z] &= 0, & (2.6)\\ \left[\frac{S_\theta F_r}{S} + (1+S_z^2)F_\theta - \frac{F_z S_z S_\theta}{S}\right] &= 0, & (2.7)\\ \left[F_r - \frac{S_\theta F_\theta}{S} - S_z F_z\right] &= \frac{J}{R_e^2}(\nabla.\hat{\mathbf{n}}), & (2.8)\end{aligned}$$

where in the core region:

$$\begin{aligned} F_r &= \frac{|\mathbf{n}|^{-2}}{R_e}\left(-PR_e + 2U_r - \frac{S_\theta}{S}\left(V_r - \frac{V}{S} + \frac{U_\theta}{S}\right) - S_z\left(U_z + W_r\right)\right),\\ F_\theta &= \frac{|\mathbf{n}|^{-2}}{R_e}\left(\frac{S_\theta}{S}\left(PR_e - \frac{2}{S}(U + V_\theta)\right) + V_r - \frac{V}{S} + \frac{U_\theta}{S} - S_z\frac{W_\theta}{S} - S_z V_z\right),\\ F_z &= \frac{|\mathbf{n}|^{-2}}{R_e}\left(U_z + W_r - \frac{S_\theta}{S}\left(\frac{W_\theta}{S} + V_z\right) + S_z R_e P - 2S_z W_z\right),\end{aligned}$$

and the stress components in the film region are obtained from the above by replacing R_e by $R_e m^{-1}$. In the normal stress equation there is a discontinuity due to surface tension as discussed previously, here we have defined a non-dimensional surface tension parameter $J = \frac{\sigma R_1 \rho_c}{\mu_c^2}$ which has been used by many authors including Hu and Joseph [9], Preziosi et al. [18] and Papageorgiou et al. [15]. The unit vector normal to the interface surface is denoted $\hat{\mathbf{n}}$ and its divergenge is,

$$\begin{aligned} \nabla.\hat{\mathbf{n}} &= \left(\frac{1}{S} - S_{zz} - \frac{S_{\theta\theta}}{S^2} + \frac{2S_\theta^2}{S^3} + \frac{S_z^2}{S} - \frac{S_{\theta\theta}S_z^2}{S^2} - \frac{S_{zz}S_\theta^2}{S^2} + \frac{2S_z S_\theta S_{\theta z}}{S^2}\right)|\mathbf{n}|^{-3};\\ |\mathbf{n}| &= \left(1 + \frac{S_\theta^2}{S^2} + S_z^2\right)^{\frac{1}{2}}.\end{aligned}$$

In addition to the above, we have the kinematic condition at the interface:

$$\frac{D}{Dt}(r - S(\theta, z, t)) = 0 \quad \Rightarrow \quad U = S_t + WS_z + \frac{V}{r}S_\theta, \quad \text{at} \quad r = S(\theta, z, t).$$

The full nonlinear equations (2.2a) to (2.3c) and boundary conditions (2.5) to (2.8) govern the incompressible core-anular flow exactly, but in general they require a numerical solution. Hu and Joseph [9] used a finite element method to solve the linearized energy problem for a general core to film ratio. In this work however, we restrict our attention to the case when the film layer is very thin compared with the radius of the core. This arises when the core fluid is separated from the pipe wall by a thin lubricating fluid layer. The method we use here essentially follows that of Papageorgiou et al. [15] by finding asymptotic solutions to the core and film fluids and hence we derive the non-linear amplitude equation which describes the evolution of the interface. In this work we investigate the effect of rotation and also allow for non-axisymmetric disturbances to the basic flow.

3 Derivation of the Evolution Equation

3.1 Thin Film Limit

We take the radius of the cylinder to be given by $r = a = 1 + \epsilon$ where $\epsilon = \frac{R_2 - R_1}{R_1} \to 0$, so that the width of the film is $O(\epsilon)$ and a lubrication type approximation is valid in that region. Suppose the general interface $r = S$ is a small deformation of the previously constant position $r = 1$, hence we write $r = S(\theta, z, t) = 1 + \delta\eta(\theta, z, t)$, where $\delta \ll \epsilon$. Although η is an unknown function, the interface conditions can be expanded in a Taylor series about the unperturbed position $r = 1$.

In the film we define a new variable y, such that:

$$r = 1 + \frac{\epsilon(1-y)}{\left(1 + \epsilon\Omega^{\frac{1}{3}}\right)}, \qquad \text{where } y = O(1) \text{ as } \epsilon \to 0. \tag{3.9}$$

In the core we define:

$$r = \frac{x}{1 + \Omega^{\frac{1}{3}}}, \qquad \text{where } x = O(1) \text{ as } \epsilon \to 0. \tag{3.10}$$

The motivation for the scaling factors involving $\Omega^{\frac{1}{3}}$ will be made clear later when we consider the limit of large rotation.

The basic velocity in the core and film is then perturbed by small amounts denoted $(\tilde{U}, \tilde{V}, \tilde{W})_{c,f}$ and the surface tension across the interface induces a jump in pressure between the core and film fluids, denoted $\tilde{P}_c$ and $\tilde{P}_f$. We substitute these quantities into the governing equations of motion and interface conditions and discard those terms which are asymptotically small as we take the thin film limit $\epsilon \to 0$. As we shall see, weakly nonlinear effects do not enter the leading order momentum equations in the film and core but in fact do so through the interface conditions leading to a nonlinear evolution equation which governs the behaviour of the interface with time.

Since we are interested in the effects of viscosity and density stratification between the two fluids, the core and film dynamics must be coupled together. This is achieved by the continuity of tangential stress, in particular, we balance the radial derivatives of both the axial and azimuthal film velocities with their corresponding counterparts in the core region. We define $\Phi = (1 + \Omega^{\frac{1}{3}})$ and $\Gamma = (1 + \epsilon\Omega^{\frac{1}{3}})$, so that $\frac{\partial}{\partial r} \sim \Phi\frac{\partial}{\partial x}$, and $\frac{\partial}{\partial r} \sim -\frac{\Gamma}{\epsilon}\frac{\partial}{\partial y}$ in the core and film respectively. If we assume that the radial gradients of the azimuthal and axial velocity components are comparable then we have:

$$\frac{\partial \tilde{W}_c}{\partial r} \sim \frac{\partial \tilde{W}_f}{\partial r} \quad \Rightarrow \quad \tilde{W}_c \sim \frac{\Gamma}{\Phi\epsilon}\tilde{W}_f, \qquad \frac{\partial \tilde{V}_c}{\partial r} \sim \frac{\partial \tilde{V}_f}{\partial r} \quad \Rightarrow \quad \tilde{V}_c \sim \frac{\Gamma}{\Phi\epsilon}\tilde{V}_f.$$

Evaluating the primary flow, (see equations (2.1a-d)) at the interface $r = \frac{x}{\Phi} = 1 + \delta\eta$, we see that the azimuthal velocity is continuous but that there is a discontinuity of size $O(\delta)$ in the axial flow, hence we require, $\tilde{W}_c \sim \delta$. The core pressure scaling is obtained from the Navier Stokes Equations (2.2a-c), and the radial velocity size is found from the equation of continuity, (2.4). The film quantities all follow similarly. The perturbed basic flow is expanded in the form:

$$\begin{aligned}
U &= U_c\frac{\delta}{\Phi} + \ldots, & U &= U_f\frac{\delta\Phi\epsilon^2}{\Gamma^2} + \ldots, && (3.11a)\\
V &= \frac{\Omega r}{R_e} + V_c\delta + \ldots, & V &= \frac{\Omega r}{R_e} + V_f\frac{\delta\Phi\epsilon}{\Gamma} + \ldots, && (3.11b)\\
W &= \overline{W}_c + W_c\delta + \ldots, & \overline{W}_f &= \overline{W}_f + W_f\frac{\delta\Phi\epsilon}{\Gamma} + \ldots, && (3.11c)\\
P &= P_J + \frac{\Omega^2 r^2}{2R_e^2} + P_c\frac{\delta\Phi^2}{R_e} + \ldots, & P &= \frac{\rho\Omega^2 r^2}{2R_e^2} + P_f\frac{\delta\Phi\Gamma}{\epsilon R_e} + \ldots && (3.11d)
\end{aligned}$$

Where for convenience we have now dropped the tilde notation and $P_J = JR_e^{-2}$ is the unperturbed pressure jump across the interface. In the view of these expressions, the kinematic condition yields,

$$\frac{\Phi\epsilon^2}{\Gamma^2}U_f = \eta_t + \overline{W}_f\eta_z + \frac{\Phi\epsilon\delta}{\Gamma}W_f\eta_z + \left(\frac{\Omega}{R_e} + \frac{\Phi\epsilon\delta V_f}{\Gamma r}\right)\eta_\theta, \quad x = \Phi(1 + \delta\eta)$$

This can, however be simplified by redefining the time as $t = \frac{\tau}{\delta}$ and making the transformations

$$z \to z - \overline{W}_{f0}t, \qquad \theta \to \theta - \frac{\Omega}{R_e}t,$$

where $\overline{W}_{f0}$ is the basic axial velocity evaluated at the unperturbed interface accurate to $O(\epsilon)$, see [15]. This is equivalent to defining a coordinate system which is moving helically down the core, that is, one which rotates with speed ΩR_e^{-1} and moves with speed $\overline{W}_{f0}$ down the axis of the cylinder. We then obtain the asymptotic size of the interface displacement δ in terms of the film width, so that $\delta = \frac{\Phi\epsilon^2}{\Gamma^2}$, and the kinematic condition reduces to:

$$U_f(y=1) = \eta_\tau - \frac{2}{m}\eta\eta_z. \tag{3.12}$$

As mentioned above, it is the surface tension which produces a jump in pressure across the interface, equation (2.8) indicates that the film pressure is of the size $\frac{J\delta}{R_e^2}$. In the light of equation (3.11d), we therefore require:

$$\frac{J\epsilon}{R_e} \sim \Phi\Gamma \sim \left(1 + \Omega^{\frac{1}{3}} + \epsilon\Omega^{\frac{2}{3}}\right). \tag{3.13}$$

By isolating this capillary stability poblem, we are effectively restricting our attention to the phyisical regimes for which the speed of the core, characterised by the Reynolds number, and the surface tension J, satisfy the above criterion. When the cylinder is fixed, or rotating with negligible speed, $\Omega \ll 1$, we see that (3.13) reduces to $\frac{\epsilon J}{R_e} = O(1)$, as found by Papageorgiou et al. [15]. To satisfy this they considered two distinct cases:

A: a slowly moving core with moderate surface tension: $R_e \sim \epsilon$ and $J \sim 1$,

B: a quicker core flow with very large surface tension: $R_e \sim 1$ and $J \sim \epsilon^{-1}$.

3.2 The Solution in the Core

After perturbing the basic flow in the core, by the quantities given by (3.11a-d), the resulting leading order equations which govern the core flow are a linear system of partial differential equations which depend on radial distance x, azimuthal angle θ and the axial coordinate z. The equations are written in terms of the new azimuthal, axial and time variable, thus for example we find that the inertial terms for the radial core momentum, transform into

$$\begin{aligned} U_t + \frac{\Phi\overline{V}}{x}U_\theta + \overline{W}U_z \quad &\rightarrow \quad \delta U_\tau + \left(\overline{W}_c - \overline{W}_{f0}\right)U_z + \left(\frac{\overline{V}}{r} - \frac{\Omega}{R_e}\right)U_\theta, \\ &= \quad \left(1 - \frac{x^2}{\Phi^2}\right)U_z + \ldots \end{aligned}$$

In order to solve the equations which govern the flow in the core we firstly employ a double Fourier transform to convert the partial differential system into one dependent

only on x and k, n the axial and azimuthal wavenumbers.
We define:

$$\left(\widehat{U}, \widehat{V}, \widehat{W}, \widehat{P}\right) = \int_{-\pi}^{+\pi} \int_{-\infty}^{+\infty} \{(U, V, W, P)\, exp\left[-i\left(kz + n\theta\right)\right]\}\, dz\, d\theta.$$

The governing equations in the core then reduce to

$$\Phi^2\left(\nabla^2\widehat{U} - \frac{\widehat{U}}{x^2} - \frac{2in\Phi\widehat{V}}{x^2}\right) - \Phi^4\widehat{P}' = \left(1 - \frac{x^2}{\Phi^2}\right) ikR_e\widehat{U} - 2\Phi\Omega\widehat{V}, \qquad (3.14a)$$

$$\Phi^3\left(\nabla^2\widehat{V} - \frac{\widehat{V}}{x^2} + \frac{2in\widehat{U}}{\Phi x^2}\right) - \frac{in\Phi^4\widehat{P}}{x} = \left(1 - \frac{x^2}{\Phi^2}\right) ik\Phi R_e\widehat{V} + 2\Omega\widehat{U}, \qquad (3.14b)$$

$$\Phi^4\left(\nabla^2\widehat{W}\right) - 2xR_e\widehat{U} + ik\Phi^4\widehat{P} = \left(1 - \frac{x^2}{\Phi^2}\right) ik\Phi^2R_e\widehat{W}, \qquad (3.14c)$$

$$\widehat{U}' + \frac{\widehat{U}}{x} + \frac{in\Phi\widehat{V}}{x} + ik\widehat{W} = 0, \qquad (3.14d)$$

where $(.)' \equiv \frac{d(.)}{dx}$ and $\nabla^2(.) \equiv \left(\frac{d^2(.)}{dx^2} + \frac{1}{x}\frac{d(.)}{dx} - \frac{n^2(.)}{x^2} - \frac{k^2}{\Phi^2}(.)\right)$.

These must be solved subject to the interface conditions:

$$\widehat{U} = 0 = \widehat{V}, \qquad \widehat{W} = \frac{2\widehat{\eta}}{m}(m-1), \qquad \text{at} \quad x = \Phi. \qquad (3.15)$$

Here $\widehat{\eta}$ has ben defined by

$$\widehat{\eta} = \int_{-\pi}^{+\pi} \int_{-\infty}^{+\infty} \{\eta(\theta, z, \tau) exp\left[-i\left(kz + n\theta\right)\right]\}\, dz\, d\theta.$$

At the axis $x = 0$ we require the velocity and pressure to remain finite.

As we shall see later, the focus of our interest in the core solution lies with the tangential stress components evaluated at the interface $x = \Phi$:

$$\frac{2\widehat{\eta}}{m}(m-1)\, N(\theta, z) = \left[\Phi^2\left(W_c\right)_{xz} + \left(U_c\right)_{zz} + x\Phi^2\left(\frac{V_c}{x}\right)_{x\theta} + \left(U_c\right)_{\theta\theta},\right] \qquad (3.16)$$

In Fourier Space this can be written:

$$\widehat{N}(n, k) = \int_{-\pi}^{+\pi} \int_{-\infty}^{+\infty} \{N(\theta, z) exp[-i(kz + n\theta)]\}\, dz\, d\theta.$$

In general the solution of equations (3.14a-d) is a numerical task, and is discussed in section (3.3). We can, however, find analytical solutions for certain specific cases.

3.2.1 Slow Core Flow Without Rotation

Consider the stability problem with moderate surface tension $J \sim 1$ as $\epsilon \to 0$, with the cylinder either fixed, or rotating with negligible speed $\Omega \ll 1$, hence $\Phi \simeq 1$. Condition (3.13) indicates that we are interested in a slow core flow, namely $R_e \sim \epsilon$. This corresponds to the equivalent axisymmetric problem (A) solved by Papageorgiou et al. [15]. The slow flow means that the inertial terms are neglected, so that in vector form the core equations reduce to $\nabla^2(\mathbf{U}_c) = \nabla P$, taking the divergence yields Stokes' Equation $\nabla^2 \hat{P} = 0$.

If we now make the transformations in Fourier space, $\hat{U}_T = \hat{U} + i\hat{V}$ and $\hat{V}_T = \hat{U} - i\hat{V}$ and define $s = kx$, then equations (3.14a) to (3.14d) reduce to

$$\begin{aligned}
\hat{P}' - \frac{n\hat{P}}{s} - k\nabla^2_{n+1}\left(\hat{U}_T\right) &= 0,\\
\hat{P}' + \frac{n\hat{P}}{s} - k\nabla^2_{n-1}\left(\hat{V}_T\right) &= 0,\\
i\hat{P} - k\nabla^2_{n}\left(\widehat{W}\right) &= 0,\\
\nabla^2_{n}\left(\hat{P}\right) &= 0,\\
\hat{U}_T' + \hat{V}_T' + \frac{(1+n)\hat{U}_T}{s} + \frac{(1-n)\hat{V}_T}{s} + 2i\widehat{W} &= 0,
\end{aligned}$$

where $(.)' \equiv \frac{d(.)}{ds}$ and $\nabla^2_n(.) \equiv \left(\frac{d^2(.)}{ds^2} + \frac{1}{x}\frac{d(.)}{ds} - \frac{n^2(.)}{s^2} - (.)\right)$.

The interface conditions become

$$\hat{U}_T = 0 = \hat{V}_T, \qquad \widehat{W} = \frac{2\hat{\eta}}{m}(m-1), \qquad \text{at } s = k,$$

Hence the solution to the core flow follows as:

$$\begin{aligned}
\hat{U}_T(k,n,x) &= A\left\{xI_n(s) - \frac{I_nI_{n+1}(s)}{I_{n+1}}\right\},\\
\hat{V}_T(k,n,x) &= A\left\{xI_n(s) - \frac{I_nI_{n-1}(s)}{I_{n-1}}\right\},\\
\widehat{W}_T(k,n,x) &= iA\left\{xI_{n+1}(s) - \frac{I_{n+1}I_n(s)}{I_n}\right\} + \hat{\eta}\frac{I_n(s)}{I_n},\\
A &= \frac{-2i\hat{\eta}kI_{n-1}I_{n+1}}{[2(2+n)I_{n-1}I_{n+1}I_n - kI_n^2I_{n-1} - kI_n^2I_{n+1} + 2kI_{n+1}^2I_{n-1}]}.
\end{aligned}$$

$I_n(s)$ is an n^{th} order modified Bessel function of s and I_n is its value at the point $s = k$. The function of stress components is then found to be:

$$\widehat{N}(n,k) = \frac{2in^2I_n\left[kI_n^2 - 2nI_{n+1}I_n - kI_nI_{n+1}\right]}{[2kI_{n+1}^2I_{n-1} - kI_n^2I_{n-1} - kI_n^2I_{n+1} + 2(2+n)I_nI_{n+1}I_{n-1}]}$$

$$+ \quad \frac{ik^2 I_{n+1}\left[kI_nI_{n-1} - 2(n-2)I_{n-1}I_{n+1} - kI_nI_{n+1}\right]}{\left[2kI_{n+1}^2I_{n-1} - kI_n^2I_{n-1} - kI_n^2I_{n+1} + 2(2+n)I_nI_{n+1}I_{n-1}\right]}$$
$$+ \quad ikn. \tag{3.17}$$

For the axisymmetric case, $n = 0$, we have an identical result to the kernel $2iN_B$ found by Papageorgiou et al. [15]:

$$\widehat{N}(0,k) = \frac{2ik^2 I_1^2}{\left[kI_1^2 - kI_0^2 + 2kI_0I_1\right]}.$$

3.2.2 Core Problem With Asymptotically Large Rotation

Let us return briefly to the perturbed Navier Stokes Equations in the core, the reason why we rescaled the core radial variable in terms of Ω will now become clear. When rotation is large, $\Omega \gg 1$, the Coriolis terms in the core momentum equations become dominant, and an analytical solution follows. To satisfy the inhomogeneous matching conditions at the interface, a non-trivial solution must be obtained near $r = 1$, we do this by increasing the size of the viscous terms in this region.

We require the following asymptotic balances:

$$\Omega V \sim R_e P_r, \qquad V_{rr} \sim R_e P_\theta \sim \Omega U, \qquad W_{rr} \sim R_e P_z, \qquad W \sim U_r \sim V.$$

Clearly then, $\frac{\partial}{\partial r} \sim \Omega^{\frac{1}{3}}$, this is the motivation behind the choice of x, as given by equation (3.10). We now let $\phi = x - 1 - \Omega^{\frac{1}{3}}$, so that the interface of the two fluids corresponds to $\phi = 0$. The effect of this rescaling is to split the core fluid layer in two; the inner region contains stationary fluid and is matched asymptotically as $\phi \to -\infty$ to the outer core region which has thickness $O(\Omega^{-\frac{1}{3}})$. It is clear, however, that the following analysis is only applicable to very thin films in which $\epsilon \ll 1$, since we require $\epsilon \ll \Omega^{\frac{1}{3}}$. Equation (3.13) indicates that for capillary stablility analysis we require $R_e \sim J\epsilon\Omega^{-\frac{1}{3}}$. Using the same method of Fourier transformation as in the previous section we find that the resulting core momentum equations are:

$$\left.\begin{aligned} 2\widehat{V} &= P', \\ \widehat{V}'' &= 2\widehat{U} + inP, \\ W'' &= ikP, \\ \widehat{U}' &= -in\widehat{V} - ikW. \end{aligned}\right\} \quad \Rightarrow \quad \widehat{V}^{(vi)} = 4k^2\widehat{V}.$$

which must be solved subject to the interface conditions:

$$\widehat{V}(\phi = 0) = 0, \quad \widehat{V}''(\phi = 0) = \frac{in\widehat{V}^{(v)}(\phi = 0)}{2k^2}, \quad \widehat{V}'''(\phi = 0) = -\frac{4ik\widehat{\eta}\,(m-1)}{m}.$$

The solution is matched to the trivial one in the inner core layer as $\phi \to -\infty$.

$$
\begin{aligned}
\widehat{V} &= A\ exp\left((2k)^{\frac{1}{3}} \frac{\left(1+i\sqrt{3}\right)}{2}\phi\right) + B\ exp\left((2k)^{\frac{1}{3}} \frac{\left(1-i\sqrt{3}\right)}{2}\phi\right) \\
&+ C\ exp\left((2k)^{\frac{1}{3}}\phi\right) \qquad k \geq 0, \\
&= A^* exp\left(-(2k)^{\frac{1}{3}} \frac{\left(1-i\sqrt{3}\right)}{2}\phi\right) + B^* exp\left(-(2k)^{\frac{1}{3}} \frac{\left(1+i\sqrt{3}\right)}{2}\phi\right) \\
&+ C^* exp\left(-(2k)^{\frac{1}{3}}\phi\right) \qquad k \leq 0, \\
A &= \frac{2\widehat{\eta}\,(m-1)}{4m\sqrt{3}(n^2+k^2)}\left[3k^2 - n^2 + i(n^2+k^2)\sqrt{3} - 4ink\right], \\
B &= \frac{2\widehat{\eta}\,(m-1)}{4m\sqrt{3}(n^2+k^2)}\left[-3k^2 + n^2 + i(n^2+k^2)\sqrt{3} + 4ink\right], \\
C &= \frac{-i\widehat{\eta}\,(m-1)}{2m},
\end{aligned}
$$

where * denotes complex conjugate.

The stress component function for $k \geq 0$ is then found to be:

$$
\widehat{N}(n,k) = \frac{(2k)^{\frac{1}{3}}\left[n + 3ik\right]}{2}, \tag{3.18}
$$

for non-positve k we have $\widehat{N}(n,-k) = \widehat{N}^*(n,+k)$.

3.3 Numerical Solution

The solution of the general core problem (3.14a-d) is a numerical task. The conditions at the interface are given by equation (3.15), but since the equations are linear, and all other conditions are homogeneous, we normalise and solve the related system which satisfies:

$$
\widehat{U}(x=\Phi) = 0 = \widehat{V}(x=\Phi), \qquad \widehat{W}(x=\Phi) = 1. \tag{3.19}
$$

Before proceeding further we must consider more closely the boundary conditions at the axis of the core $x = 0$. These have received much attention by many authors including, Batchelor and Gill [1] and Preziosi et al. [18], and so we briefly summarise their findings here. The physical quantities $(U, V, W, P)_c$ should remain finite at the origin and must also be single valued there, that is, independent of θ (see Joseph [11]).

$$\begin{aligned} n = 0: &\qquad \widehat{U}(0) = \widehat{V}(0) = \widehat{W}'(0) = 0, \\ n = 1: &\quad \widehat{U}(0) + \widehat{V}(0) = \widehat{W}(0) = \widehat{P}(0) = 0, \\ n \geq 2: &\quad \widehat{U}(0) = \widehat{V}(0) = \widehat{W}(0) = \widehat{P}(0) = 0. \end{aligned} \tag{3.20}$$

The sixth order system of equations exhibit a singularity at the origin and admit three irregular solutions there which are to be discarded. We find the three remaining regular, independent power series solutions near $x = 0$ which satisfy the above conditions for the relevant azimuthal wavenumber n. Recurrence relations for these series are included for completeness in Appendix A.

In the usual way, pressure is eliminated from equations (3.14a-d) so that they may be solved by means of a fourth order Runge Kutta scheme. The first N terms in the series are evaluated at $x = \xi \ll 1$ so that a numerical solution can be found by marching in x with step length h. A linear combination of the three solutions is then found to satisfy the interface conditions (3.19) at $x = \Phi$. The size of the starting point ξ, the number of terms in the series N and the step length h are chosen so that this is achieved to within a required tolerance. The function of tangential stress components $N(n,k)$ is thus found to high degree of accuracy. The results are discussed in the next chapter.

3.4 The Solution in the Film

The film layer has thickness $O(\epsilon)$ and lubrication theory is appropriate there. We note that the film is thin enough to satisfy $\Omega \ll \epsilon^{-3}$, as discussed previously, and that the equations are not directly affected by rotation at leading order since the Coriolis terms are negligible, and $\Gamma \simeq 1$. The solution which satisfies no slip at the cylinder wall ($y = 0$) is as follows:

$$P_f = P_f(\theta, z, \tau), \tag{3.21a}$$

$$W_f = \frac{y^2}{2m} P_{f_z} + y A(\theta, z, \tau), \tag{3.21b}$$

$$V_f = \frac{y^2}{2m} P_{f_\theta} + y B(\theta, z, \tau), \tag{3.21c}$$

$$U_f = \frac{y^3}{6m}\left(P_{f_{\theta\theta}} + P_{zz}\right) + \frac{y^2}{2}\left(A_z + B_\theta\right). \tag{3.21d}$$

The functions A and B which are independent of y are found by substituting the core solutions into the equations of tangential stress continuity:

$$(V_f)_y = \frac{-1}{m}\left[\left(\frac{V_c}{x}\right)_x x + \frac{(U_c)_\theta}{\Phi^2}\right], \qquad (W_f)_y = \frac{-1}{m}\left[(W_c)_x + \frac{(U_c)_z}{\Phi^2}\right],$$

evaluated at $y = 1$ and $x = \Phi$.

3.5 Derivation of the Evolution Equation

Now that we have solved the velocity and pressure fields for both the core and film layers, we can derive the equation which governs the time evolution of the small disturbance to the fluid-fluid interface. The kinematic condition is given by equation (3.12) and the continuity of normal sress is

$$P_f(y=1) = T_0\left(\eta + \eta_{zz} + \eta_{\theta\theta}\right) + R_0(1-\rho)\eta. \tag{3.22}$$

Here we have defined the surface tension parameter to be $T_0 = \frac{J_\epsilon}{R_e \Phi \Gamma}$, an $O(1)$ constant as $\epsilon \to 0$. The term involving $R_0 = \frac{\epsilon \Omega^2}{\Phi \Gamma R_e}$ is due to the density stratification of the two fluids and it arises as a result of the Taylor expansion of the basic pressure about the constant interface position. Clearly, when the rotation Ω is very large this term will dominate equation (3.22), this means that the density stratification has a crucial effect on the evolution of the interface. We therefore restrict ourselves to flows in which the density difference between the two fluids is small, so that the term $(1-\rho)T_0$ will always be at most an order one quantity. As discussed previously, we have chosen to neglect the effect of gravity for this problem, and so the role of density stratification is not as significant as the difference between the viscosities of the two fluids. The effect of gravity has been solved for the linear problem by Chen et al. [4] and Georgiou, Maldarelli, Papageorgiou and Rumschitzki [6].

By equating expressions for the radial film velocity U_f given by equations (3.12) and (3.21d), evaluating them at the unperturbed interface radius y=1 and substituting for A and B, we obtain the evolution equation in the form:

$$\begin{aligned}\eta_\tau &- \frac{2}{m}\eta\eta_z + \frac{T_0}{3m}\left(\eta_{zz} + \eta_{\theta\theta} + \eta_{zzzz} + \eta_{\theta\theta\theta\theta} + 2\eta_{zz\theta\theta}\right) \\ &+ (1-\rho)\frac{R_0}{3m}\left(\eta_{zz} + \eta_{\theta\theta}\right) + \frac{(m-1)\,\eta N(\theta,z)}{m^2\Phi^2} = 0. \end{aligned} \tag{3.23}$$

Making the fluids the same density and considering the axisymmertic case $\partial_\theta = 0$, without rotation, reduces this identically to the evolution equation found by Papageorgiou et al., namely the Kuramoto-Sivashinsky equation together with the additional term $N(\theta, z)$ which is due to the dynamics of the core fluid, coupled by the viscosity stratification.

4 Solution of the Evolution Equation

The time evolution of the interface between the core and film fluids is governed by equation (3.23), which may be written in the form:

$$\eta_\tau - \frac{2}{m}\eta\eta_z + \frac{(1-\rho)\,R_0}{3m}\nabla^2(\,\eta\,) + \frac{T_0}{3m}\nabla^2\left(\eta + \nabla^2\eta\right) + \frac{(m-1)}{4\pi^2 m^2\Phi^2} \times$$
$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \widehat{N}(n,k) \int_{-\pi}^{+\pi}\int_{-\infty}^{+\infty} \eta(z_1,\theta_1,\tau)\, e^{[ik(z-z_1)+in(\theta-\theta_1)]} dz_1 d\theta_1 dk dn$$

where: $\nabla^2\eta = (\partial_{zz} + \partial_{\theta\theta})\,\eta$.

The second derivative terms are linearly destabilizing and compete with the stabilizing fourth derivates. It is this competition which keeps the nonlinear solutions from breaking up, although there is as yet no formal proof that the solutions must remain bounded for all time when we have dependence on this additional angular coordinate θ. Before considering the diverse array of solutions offered by this nonlinear partial differential equation, it is firstly instructive to examine the simpler problem where the nonlinear term is absent. We look for solutions which are periodic in both the axial and azimuthal directions. Clearly η must be single valued in θ so that $\eta(\theta,z,t) = \eta(\theta+2\pi,z,t)$, however the length of the period in z is not determined and is indeed an integral part of the dynamics of the solution of the full nonlinear equation.
We normalise the axial coordinate so that solutions, η, are also 2π periodic in z, we then look for modes of the form $\eta \sim \exp\left[ct + i\lambda^{\frac{1}{2}}kz + in\theta\right]$, the linearized dispersion relation is:

$$c_r = \alpha\left(\lambda k^2 + n^2\right) + \left(\lambda k^2 + n^2\right)\left(1 - \lambda k^2 - n^2\right) - \gamma\widehat{N}_r\left(\lambda^{\frac{1}{2}}k, n\right).$$

The parameter $\alpha = (1-\rho)\,R_0 T_0^{-1}$ represents the effect of the density stratification and rotation of the fluids, whilst the surface tension term has been normalized to unity by rescaling the time derivative. The core and film dynamics are coupled by the difference in viscosity between the two fluid layers and this enters the dispersion relation through the term $\gamma N_r = \frac{3(m-1)}{m\Phi^2 T_0} N_r$, where the subscript r denotes the real part. The sign of γ is made negative when the core fluid is more viscous than the film.

In accord with the usual theory of temporal stability, c is complex valued and a positive real part indicates exponential linear growth and hence instability. When both the viscosities and the densities are continuous across the interface, $(\alpha \equiv \gamma \equiv 0)$, we can see from Figure 1 that the axisymmetric, $n = 0$ mode has a positive growth rate and is the most unstable disturbance, in fact the azimuthal modes become progressively more stable with increasing wavenumber n. When the cylinder rotates, the density ratio of the two fluids can alter the stability of the CAF. If the core fluid is more dense than the film, ($\alpha > 0$), the non-axisymmetric perturbations become destabilized, with the $n = 1$ mode being most dangerous, see figure 2a. For the inverse arrangement, that is with the lighter fluid in the core then all disturbances are stabilized due to the centripetal effects as shown by figure 2b. This is in agreement with the conclusions of Hu and Joseph [9].

More importantly we wish to quantify the influence of the core fluid dynamics on the interfacial disturbance. For a given surface tension and rotation we use the numerical results obtained from section (3.3) to calculate the tangential stress at the interface and hence the term $N(k, n)$. In addition to the numerical solution of the core problem we can use the analytical results of sections (3.2.1) and (3.2.2). In the former case we have a slow core flow and negligible rotation leading to a solution of the Stokes' problem for which we obtained a term $N(k, n)$ which is imaginary, and hence has a purely dispersive effect on the evolution. For the simplified axisymmetric case this reduces to the kernel $2iN_B(k)$ found by Papageorgiou et al. [15]. Although the linearized solutions are unaltered by this term, it does play an important role in the nonlinear evolution.

In the presence of larger rotation we obtain the much simpler kernel :

$$\widehat{N}(k,n) = \frac{(2k)^{\frac{1}{3}}\left[n + 3ik\right]}{2}, \qquad k \geq 0,$$

$$\widehat{N}(k,n) = \frac{(2k)^{\frac{1}{3}}\left[n - 3ik\right]}{2}, \qquad k \leq 0.$$

Figure 3 shows that when the core fluid is more viscous than the film, ($m < 1$ and hence $\gamma < 0$), the interface is stabilized according to linear theory. When the core is less viscous, for example the model representing the flow of lubricated crudes through rotating drills, we see from Figure 4 that the first non-axisymmetric mode is unstable and the linear analysis predicts exponential growth of the interfacial disturbances.

This clearly is an unphysical situation and we must turn to the nonlinear theory for an explanation of such effects. Practical observations of such bicomponent rotating flows indicate that stable traveling waves may exist as well as the chaotic mixing of the transported fluid and its less viscous lubricant.

4.1 Numerical Method

For the solution of the full nonlinear evolution equation we follow the method used in the extensive numerical work done by Papageorgiou and Smyrlis [16] and [17] in their comprehensive investigations of the route to chaos taken by spatially periodic solutions of the Kuramoto-Sivashinsky equation.

$$U_t + UU_x + U_{xx} + \lambda U_{xxxx} = 0,$$

$$U(x,0) = U_0(x), \qquad U(x+2\pi, t) = U(x,t).$$

The numerical method used in this work essentially follows theirs and consequently we need only summarise it briefly and discuss its extension to include the additional terms which represent the azimuthal coordinate. As described above we look for solutions of the evolution equation which have period 2π in both the azimuthal and axial directions having made the transformation of the form $z \rightarrow \lambda^{\frac{1}{2}} z$.

We used a pseudo-spectral method in which the spatial dependence of η is represented in wave number space by employing double Fourier transforms. The initial condition used by Papageorgiou and Smyrlis was given by $U_0 = -\sin(x)$, here we considered two separate initial value problems in which η is a prescribed function of z and θ of the form:

$$\eta = \sum_{k=1}^{\infty} \sum_{n=0}^{\infty} \eta_{k,n} \sin(kz) \cos(n\theta), \tag{4.24}$$

$$\eta = \sum_{k=1}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \eta_{k,n}^{(c)} \cos(kz + n\theta) + \eta_{k,n}^{(s)} \sin(kz + n\theta) \right\}. \tag{4.25}$$

A traditional Galerkin approximation was then used to produce the following truncated system of ODE's for $1 \leq k \leq K$ and $-N \leq n \leq N$. These are solved in a time marching sequence using a Runge-Kutta scheme.

$$\frac{d}{dt}\eta_{k,n} = \Lambda_{k,n}\eta_{k,n} + F_{k,n}(\eta_{k,n})$$

The term linear $\Lambda_{k,n}$ arises from the Fourier transform of the second and fourth derivative terms together with the kernel, as discussed above. The nonlinear contribution $\eta\eta_z$ is found by manipulating the series expressions for η, these may then be expressed as a combination of the initial spatial profile for each of the two cases considered. For η of the from (4.24), the term $F_{k,n}$ can be written as:

$$F_{k,n} = \frac{1}{4} \left\{ \sum_{\substack{k_1+k_2=k \\ n_1+n_2=n}} + \sum_{\substack{k_1+k_2=k \\ |n_1-n_2|=n}} + \sum_{\substack{k_1-k_2=k \\ n_1+n_2=n}} \right.$$

$$
\left. - \sum_{\substack{k_2-k_1=k \\ n_1+n_2=n}} + \sum_{\substack{|k_1+k_2|=k \\ |n_1-n_2|=n}} - \sum_{\substack{k_2-k_1=k \\ |n_1-n_2|=n}} \right\} k_2 \eta_{k_1,n_1} \eta_{k_2,n_2} \sin(kz) \cos(n\theta) .
$$

For (4.25) we have:

$$
\begin{aligned}
F_{k,n} &= \frac{k}{2} F_{k,n}^{(c)} \cos(kz+n\theta) - \frac{k}{4} F_{k,n}^{(s)} \sin(kz+n\theta) \\
F_{k,n}^{(c)} &= \sum_{\substack{k_1+k_2=k \\ n_1+n_2=n}} \eta_{k_1,n_1}^{(c)} \eta_{k_2,n_2}^{(s)} + \sum_{\substack{k_1-k_2=k \\ n_1-n_2=n}} \left[\eta_{k_2,n_2}^{(c)} \eta_{k_1,n_1}^{(s)} - \eta_{k_1,n_1}^{(c)} \eta_{k_2,n_2}^{(s)} \right] \\
F_{k,n}^{(s)} &= \sum_{\substack{k_1+k_2=k \\ n_1+n_2=n}} \left[\eta_{k_1,n_1}^{(c)} \eta_{k_2,n_2}^{(c)} - \eta_{k_1,n_1}^{(s)} \eta_{k_2,n_2}^{(s)} \right] + \sum_{\substack{k_1-k_2=k \\ n_1-n_2=n}} 2 \left[\eta_{k_1,n_1}^{(c)} \eta_{k_1,n_1}^{(c)} + \eta_{k_1,n_1}^{(s)} \eta_{k_2,n_2}^{(s)} \right]
\end{aligned}
$$

Throughout the numerical solution we compute the energy, (or L^2-norm), the Fourier modes and a phase-plane map, which is the energy of the system against its time derivative. An adaptive time step was used so that the error was minimised and the solution remained stable. For a full discussion of these points see [16].

4.2 Numerical Results

Let us consider first the solutions obtained when the densities and viscosities are the same for both the core and film fluids, so that $\rho = 1$, and $m = 1$. We used a prescribed function of the form (4.24), with initial conditions which contained both axisymmetric and non-axisymmetric modes. When $\lambda \geq 1$ constant states are obtained, as λ decreases below unity however an array of nonlinear phenomena are observed. The conclusion of the lengthy computational studies undertaken in this work is that the azimuthal modes decay rapidly and only the axisymmetric, $n = 0$, modes are significant. Consequently the results obtained here are closely related to those found by Papageorgiou and Smyrlis [16] and [17] for the solution of the standard Kuramoto-Sivashinsky equation.

Parameter Range	**Description**
$1 \leq \lambda < \infty$	Constant States
$0.248 \leq \lambda \leq 1$	Fully Modal Steady Attractor
$0.0755 \leq \lambda \leq 0.247$	Bimodal Steady Attractor
$0.034 \leq \lambda \leq 0.075$	Fully Modal Steady Attractor
$\lambda \leq 0.033$	Mostly Chaotic Oscillations

Table 1. Summary of the Solutions with matched densities and viscosities.

The results illustrated in table 1. are not to be considered as a full description of the solutions of this equation, to do this we would require many hundreds more numerical experiments. Moreover, we seek to indicate that the rough estimates to the window bounds shown above are consistent with those for the standard KS equation and that no new behaviour has been discovered. When $\lambda \leq 0.033$ the solutions become chaotic, although a closer numerical analysis indicates that time-periodic behaviour may also exist for sub-windows within this region of parameter space. As λ decreases, more accurate, and hence longer, numerical experiments must be made to define precisely the behaviour as the solution undergoes a transition towards full chaos.

Similar numerical studies were undertaken using a solution of the form given by equation (4.25). In the absence of a viscousity differnce between the fluid layers we see similar solutions to those outlined above, in fact if the cosine behaviour is dropped the solutions are identical.

Figure 5(a) shows a plot of energy against time for a particular case in the chaotic window, namely $\lambda = 0.02$. We note that although the solution remains bounded for all time, the structure is indeed that of fully chaotic oscillations as the phase-plane diagram, as figure 5(b) indicates.

By preserving the modes $\eta_{k,n}^{(c)}$ and $\eta_{kn,}^{(s)}$ at successive time intervals, we may visualise the evolution of the interfacial disturbance. The chaotic solution as a function of periodic axial variable z, and fixed azimuthal coordinate θ, is shown by figure 6. The solution is shown at successive time intervals, and these are plotted with a displacement of 1 unit.

We now consider the evolution of the interface when the two fluid layers are stratifed by a difference in their kinematic viscosities, so that we introduce the kernel term proportional to $N(\theta, z)$. This additional term given by the coupled dynamics of the core region can alter the stability of the interface considerably. As we have discussed previously, this term is found either numerically for a general core speed, surface tension and rotation or may be given by the analytical work in (3.2.1) and (3.2.2).

In this work we present the results for a quickly rotating core-annular fluid with a relatively low coefficient of surface tension between the two fluids layers. In this case the kernel $\widehat{N}(n,k)$ is complex valued and its imaginary part is proportional to $k^{\frac{4}{3}}$. The real part allows for linear growth or decay of non-axisymmetric disturbences, as discussed above.

We have already seen that in the absence of viscosity stratification the interface may become chaotic in time, an important question then is what effect does the this viscosity difference make when the full nonlinear evolution equation is solved? For a less viscous core layer, how does the nonlinear behaviour cope with the exponential linear growth of the non-axisymmetric disturbances to the interface?

The results of our numerical investigations are that the viscousity difference organises the otherwise chaotic motion of the interface into periodic waves which travel in both the axial and azimuthal directions. After only a short amount of time the energy, (the integral of η^2 over its domain), becomes constant. The solution is illustrated by figures 7 and 8 which show the axial and azimuthal travelling waves respectively. In Figure 7 we have displaced the graphs vertically by 2 units and we have included three periods so that the motion of the wave can be visualised more clearly. When analysing these solutions one must recall that this is in fact a wave superposed on a coordinate system which already moves helically down the core. Figure 8 clearly indicates that this new interfacial profile is indeed non-axisymmetric. Again we include three periods and have a vertical displacement of 15 units.

Upon increasing the ratio of the core viscosity to that of the film fluid, the amplitude of these waves is increased, but does not grow exponentially in time as the results of the linear theory given here and that of Hu et al. [10] suggests. If the core is less viscous than the film fluid the linearly stabilizing influence could ultimately yield the trivial solution.

5 Conclusions

In this work we have studied the linear and nonlinear behaviour of a rotating two-phase core-annular flow and identified the important mechanisms which govern the instability of the fluid-fluid interface.

When a rotating fluid encapsulates a more dense core, the centripetal forces lead to the growth of non-axisymmetric disturbances to the concentric arrangement. More importantly perhaps, a viscosity difference between the fluids also leads to an unstable non-axisymmetric regime. The linear theory predicts that these three dimensional disturbances will grow exponentially in time.

The numerical solutions of the weakly nonlinear evolution equation derived here indicate that in fact travelling waves are obtained. These waves are distortions of the interface and move helically down the core. Their amplitude depends upon the difference in the viscosities of the two fluids.

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A Power Series Coefficients

In section 3.3 we discussed the numerical solution of the core problem and seek a power series solution to equations (3.14a-d) near the axis $x = 0$. We look for a solution of the form:

$$U = \sum_{j=0}^{\infty} U_j r^j, \qquad V = \sum_{j=0}^{\infty} V_j r^j,$$

$$W = \sum_{j=0}^{\infty} W_j r^j, \qquad P = \sum_{j=0}^{\infty} P_j r^j.$$

In the light of the conditions given by (3.20) must consider each case separately.

A.1 Axisymmetric Modes n=0

W_0, P_0 and V_1 are arbitrary; U_0, W_1, V_0, P_1, U_2, V_2, W_3 and P_3 are all zero.

$$\begin{aligned}
U_1, &= -\frac{1}{2} ikW_0, \\
U_3, &= -\frac{1}{4} ikW_2, \\
V_3, &= \frac{1}{8\alpha}\left(2\Omega U_1 + ik\alpha R_e V_1 - k^2 V_1\right), \\
W_2, &= \frac{1}{4}\left(k^2 W_0 + ik\alpha^2 P_0 + ikR_e W_0\right), \\
P_2, &= \frac{1}{2\alpha^4}\left(8\alpha^2 U_3 - ikU_1 - k^2 U_1 + 2\alpha\Omega V_1\right).
\end{aligned}$$

For $j = 4, 5, 6, \ldots$,

$$\begin{aligned}
U_j, &= -\frac{ikW_{j-1}}{(j+1)}, \\
V_j, &= \frac{1}{\alpha^2 (j^2 - 1)}\left[\alpha^2 k^2 V_{j-2} + 2\alpha\Omega U_{j-2} + ik\alpha^2 R_e V_{j-2} - ikR_e V_{j-4}\right],
\end{aligned}$$

$$
\begin{aligned}
W_j, &= \frac{1}{\alpha^2 j^2}\left[\alpha^2 k^2 W_{j-2} + ik\alpha^4 P_{j-2} - 2R_e U_{j-3}\right.\\
&+ \left. ikR_e\alpha^2 W_{j-2} - ikR_e W_{j-4}\right],\\
P_j, &= \frac{1}{\alpha^6 j}\left[2\alpha^3\Omega V_{j-1} - ik\alpha^2 R_e U_{j-1} + ikR_e U_{j-3}\right.\\
&+ \left. j(j+2)\alpha^4 U_{j+1} - \alpha^2 k^2 U_{j-1}\right].
\end{aligned}
$$

A.2 Non-Axisymmetric Modes

For the first non-axisymmetric mode $n = 1$: W_1, P_1 and U_0 are arbitrary, W_0, P_0, U_1, V_1, W_2 and P_2 are all zero.

$$
\begin{aligned}
U_2, &= \frac{1}{8\alpha^2}\left(k^2 U_0 - 2i\alpha 3\Omega U_0 + ikR_e\alpha^2 U_0 + \alpha^6 P_1 - 2ik\alpha W_1\right),\\
V_2, &= 3iU_2 - kW_1,\\
W_3, &= \frac{1}{8\alpha^4}\left(\alpha^2 k^2 W_1 + ik\alpha^6 P_1 - 2R_e U_0 + ik\alpha^2 R_e W_1\right),\\
U_3, &= 0,\\
V_3, &= 0.
\end{aligned}
$$

For subsequent modes $n \geq 2$: W_n, P_n and U_{n+1} are arbitrary, U_0, V_0, W_0, P_0,...,U_n, V_n, P_{n-1}, W_{n-1} are all zero.

$$
\begin{aligned}
V_{n+1}, &= \frac{i\,(n+2)\,U_{n+1}}{n} - \frac{kW_n}{n}\\
W_{n+1}, &= 0,\\
P_{n+1}, &= 0.
\end{aligned}
$$

For $j = n+2, n+3, \ldots$,

$$
\begin{aligned}
U_j, &= \frac{\alpha^{-4}}{\left[(j+1)^2 - n^2\right]}\left[\alpha^2 k^2 U_{j-2} - 2ikW_{j-1} + (j-1)\,\alpha^6 P_{j-1}\right.\\
&- \left. 2\alpha^3\Omega V_{j-2} + ikR_e\left(\alpha^2 U_{j-2} - U_{j-4}\right)\right],\\
V_j, &= \frac{1}{\alpha n}\left[i\,(j+1)\,U_j - kW_{j-1}\right],\\
W_j, &= \frac{1}{\alpha^4\left(j^2 - n^2\right)}\left[\alpha^2 k^2 W_{j-2} + ik\alpha^4 P_{j-2} + ikR_e\left(\alpha^2 W_{j-2} - W_{j-4}\right)\right.\\
&- \left. 2R_e U_{j-3}\right],\\
P_j, &= \frac{1}{\alpha^5 f}\left[2n^2 + \left(\left(j^2 + 2j - n^2\right)(j+2)\right)\left(ikR_e\left(\alpha^2 U_{j-1} - U_{j-3}\right)\right.\right.
\end{aligned}
$$

$$
\begin{aligned}
& - \quad 2\alpha^3\Omega V_{j-1} - 2ik\alpha^4 W_j \\
& + \quad \alpha^2 k^2 U_{j-1}\Big) + \left((j+2)^2 - n^2\right)\left(nk\alpha R_e\left(V_{j-3} - \alpha^2 V_{j-1}\right) + \Omega U_{j-1}\right. \\
& + \quad \left. 2i\alpha^2 ink^2\alpha^3 V_{j-1} + \left(j^2+2j-n^2\right) ik\alpha^4 W_j\right)\Big], \\
f(j) &= \left((j+2)^2 - n^2\right) - \left(j^2+2j\right)\left(j^2+2j-n^2\right) + 2n^2 j + n^2.
\end{aligned}
$$

References

[1] Batchelor G.K. and Gill A.E. Analysis of the Stability of Axisymmetric Jets (1962) J. Fluid Mech. Vol. 24 pp 529-551

[2] Blennerhassett P.J. The Generation of Waves by Wind (1980) Phil. Trans. Roy. Soc. (A) Vol. 298 pp 451-493

[3] Charles M.E., Govier G.W. and Hodgson G.W. The Horizontal Pipeline Flow of Equal Density of Oil-Water Mixtures (1966) Can. J. Chem. Engng Vo. 39 pp 17-36

[4] Chen K., Bai R. and Joseph D.D. Lubricated Pipelining: Stability of Core Annular Flow in Vertical Pipes (1990) J. Fluid Mech. Vol. 214 pp 251-286

[5] Frenkel A.L., Babcin A.J., Livich B.G., Shlang T. and Sivashinsky G.I. (1987) J.Colloid. Interface Sci. Vol. 115 pp 225-233

[6] Georgiou E., Maldarelli C., Papageorgiou D.T. and Rumschitzki D.S., Linear Stability of Core-Annular flow (1992) J. Fluid Mech. Vol. 243 pp 653-677

[7] Hickox C. Instability Due to Viscosity and Density Stratification in Axisymmetric Pipe Flow (1971) Phys. Fluids Vol. 14 pp 251-262

[8] Hooper A.P. and Boyd W.G.C. Shear Flow Instability at the interface between two viscous fluids (1983) J. Fluid Mech. Vol. 128 pp 507-528

[9] Hu and D.D.Joseph Lubricated Pipelining: Stability of Core-Annular Flow Part 2 (1989) J. Fluid. Mech. Vol. 205 pp 359-396

[10] Hu and D.D.Joseph Stability of Core-Annular Flow in a Rotating Pipe (1989) Phys. Fluids (A) Vol. 1 pp 1677-1585

[11] Joseph D.D. Stability of Fluid Motions (1976) Springer, Berlin Vol. 2

[12] Joseph D.D., Renardy M. and Renardy Y. Instability of the Flow of Immiscible Liquids in a Pipe (1984) J. Fluid Mech. 141 pp 309-317

[13] Lamb H. Hydrodynamics (1936) Cambridge University Press

[14] Miesen R., Beijnon G., Duijvestijn P.E.M., Oliemans R.V.A. and Verheggen T. Interfacial Waves in Core-Annular Flow (1992) J. Fluid Mech. 238 pp 97-117

[15] Papageorgiou D.T., Maldarelli C. and Rumschitzki D.S., Nonlinear Interfacial Stability of Core-Annular flow (1990) Phys. Fluids (A) Vol. 2 Part 3 pp 340-352

[16] Papageorgiou D.T. and Smyrlis Y.S., The Route to Chaos for the Kuramoto-Sivashinsky Equation (1991) Theoret. Comput. Fluid Dynamics Vol. 3 Part 1 pp 15-42

[17] Papageorgiou D.T. and Smyrlis Y.S., Predicting Chaos for Infinite-Dimensional Dynamical Systems: The Kuramoto-Sivashinsky Equation, A Case Study (1991) Proc. Nat. Acad. Sciences, USA Vol. 88 No. 24 pp 11129-11132

[18] Preziosi L., Chen K., and Joseph D.D. Lubricated Pipelining Stability of Core-Annular Flow (1989) J. Fluid Mech. 201 pp 323-356

[19] Renardy Y. Instability at the Interface Between Two Shearing Fluids in a Channel (1985) Phys. Fluids Vol. 28 pp 3441-3443

[20] Yih C.S. Instability Due to Viscosity Stratification (1967) J. Fluid Mech. Vol. 27 pp 337-352

Fig. 1 : Linearised Evolution: Equal Densities and Viscosities

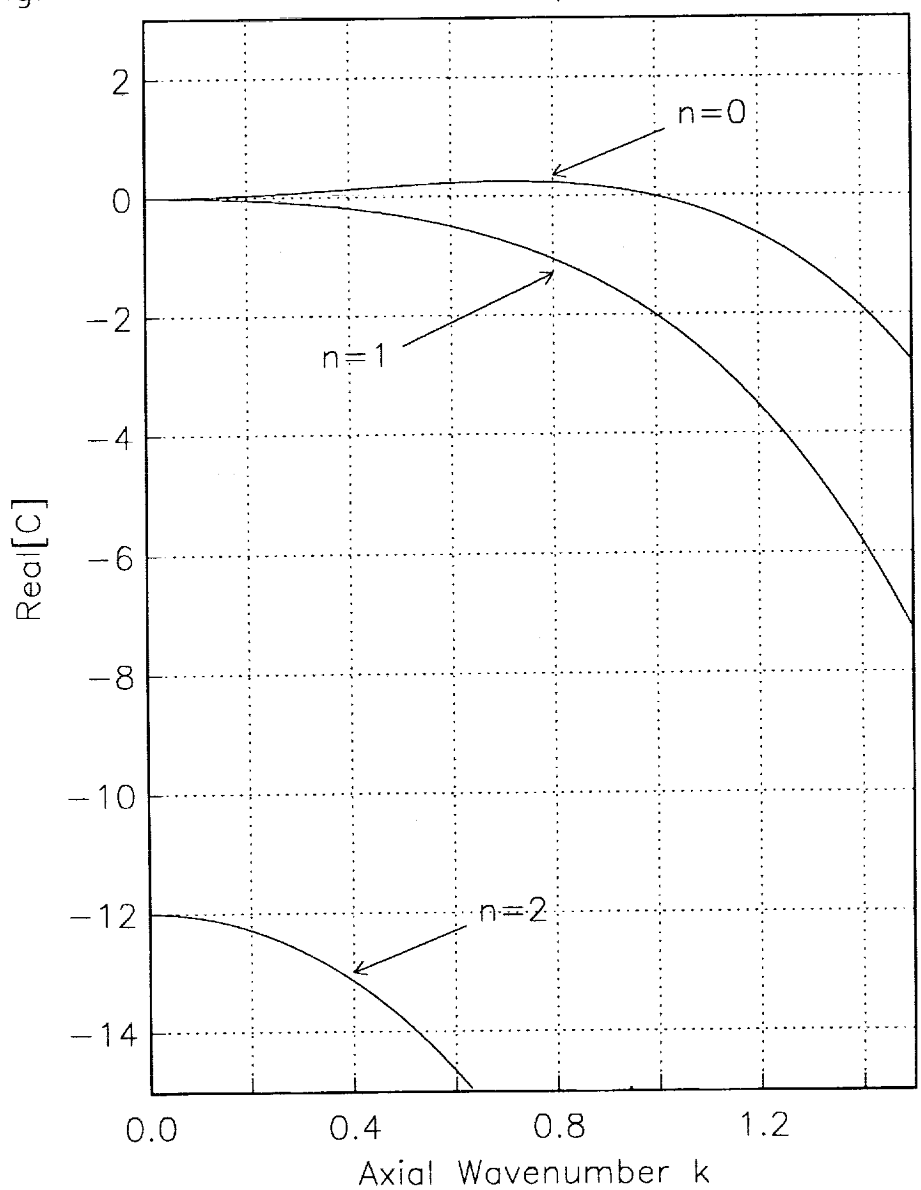


Fig.2a : Linearised Evolution : A Heavier Core Fluid

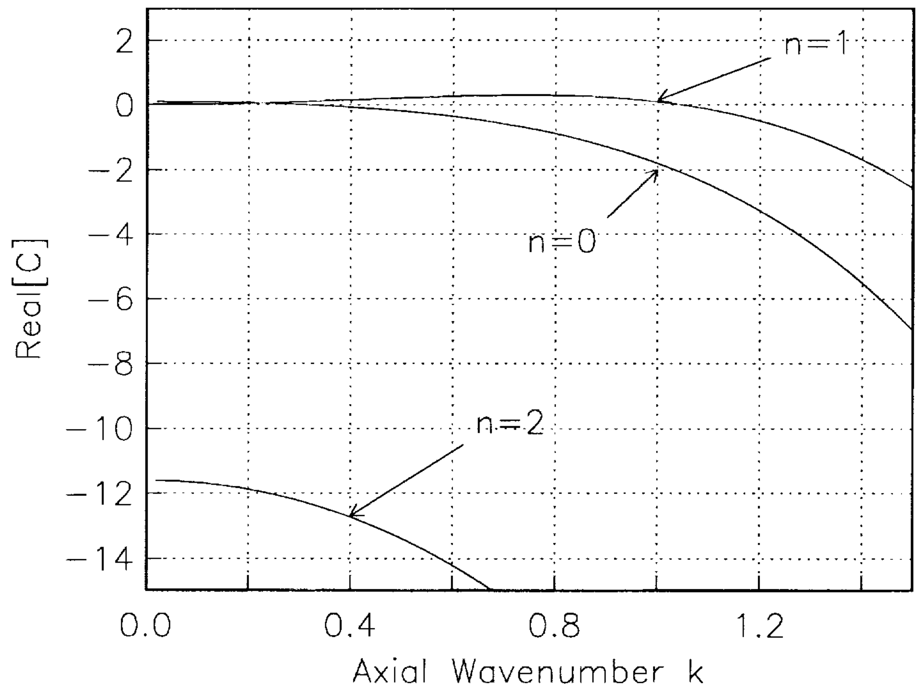


Fig.2b : Linearised Evolution : A Lighter Core Fluid

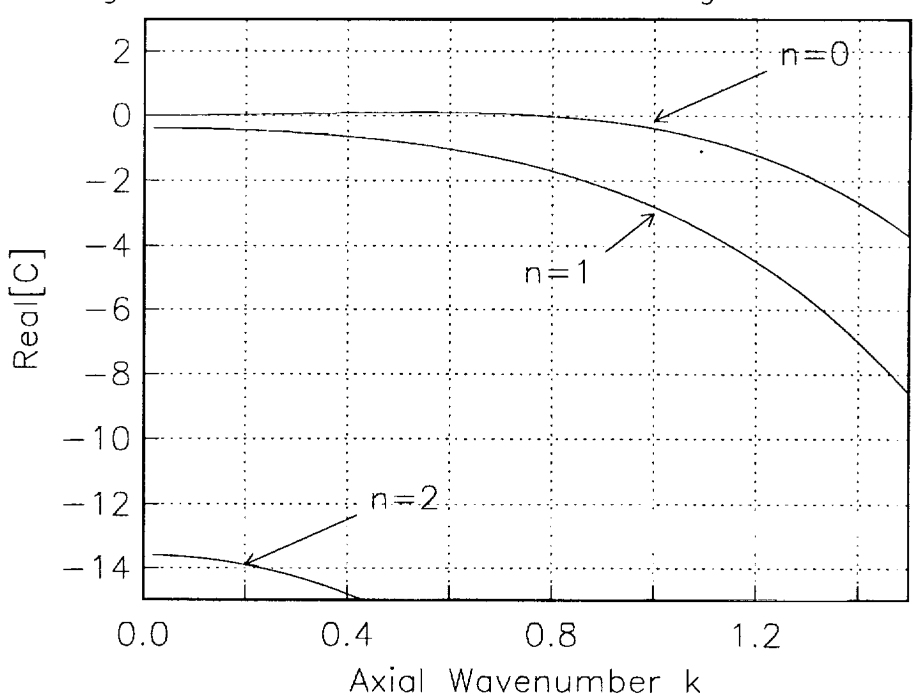


Fig.3 : Linearised Evolution : Large Rotation & Less Viscous Core

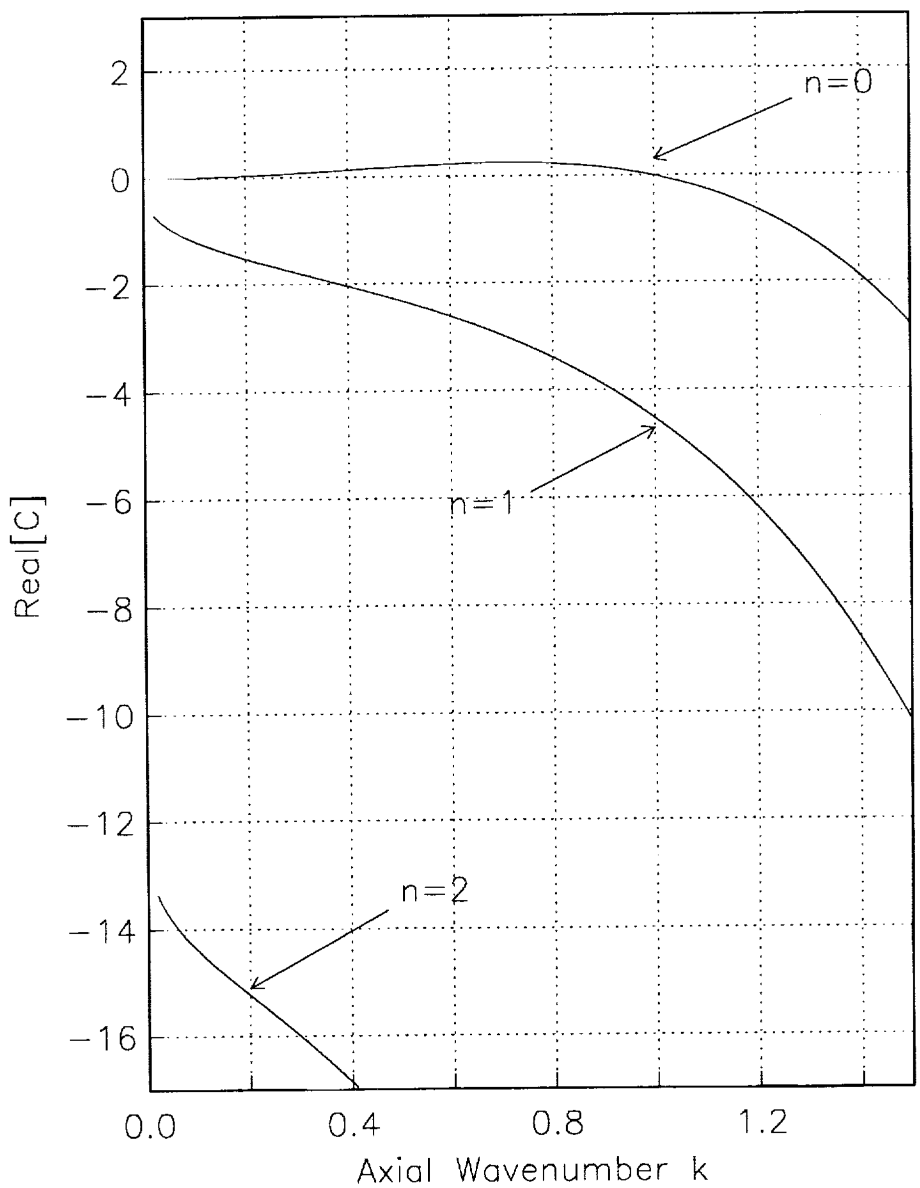


Fig.4 : Linearised Evolution : Large Rotation & More Viscous Core

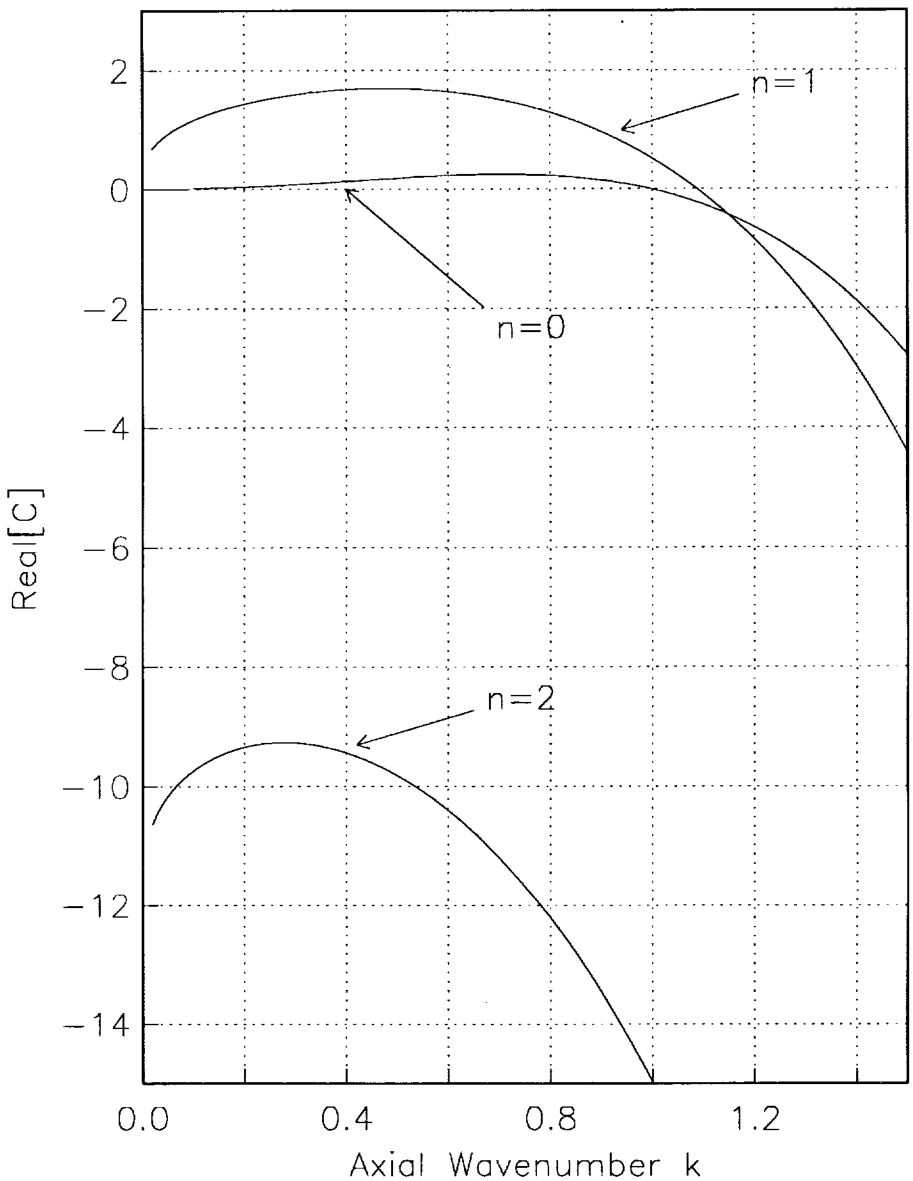


Fig. 5(a) Energy–Time Graph : $\lambda=0.02$, Equal Densities & Viscosities

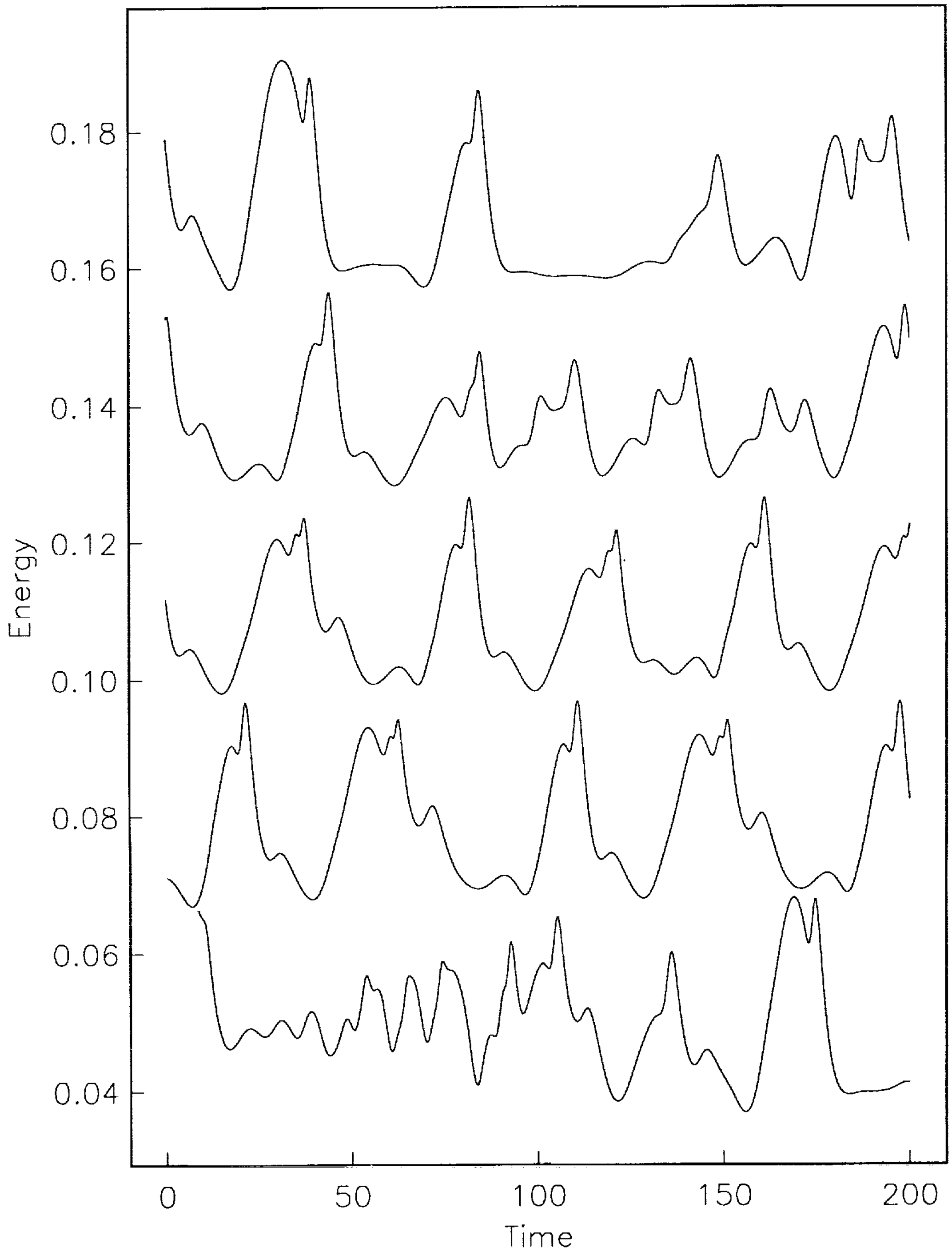


Fig. 5(b) Phase Plane : λ=0.02, Equal Viscosities and Densities.

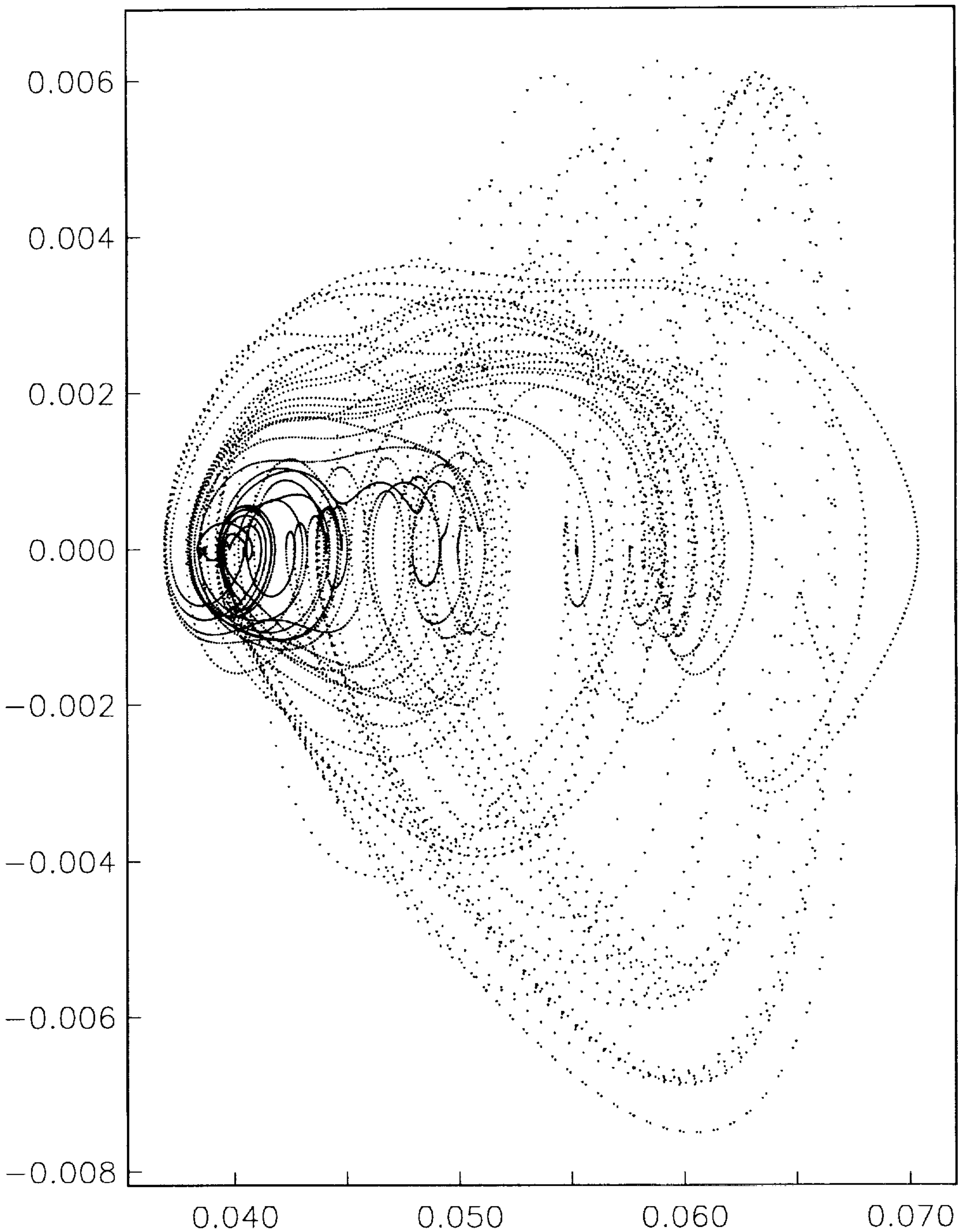


Fig. 6 Solution η in Z with Fixed ϑ

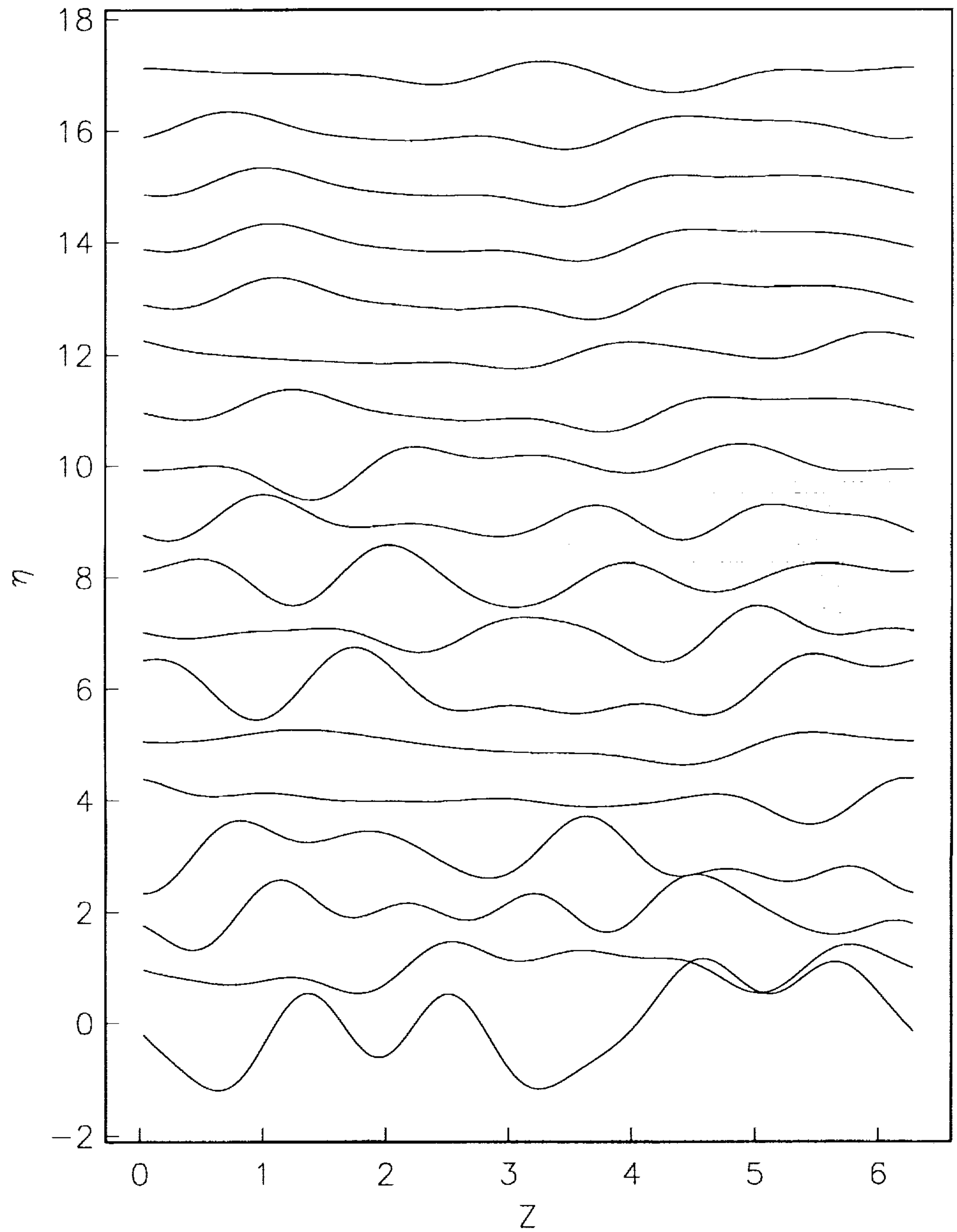


Figure 7 Solution With Viscosity Difference

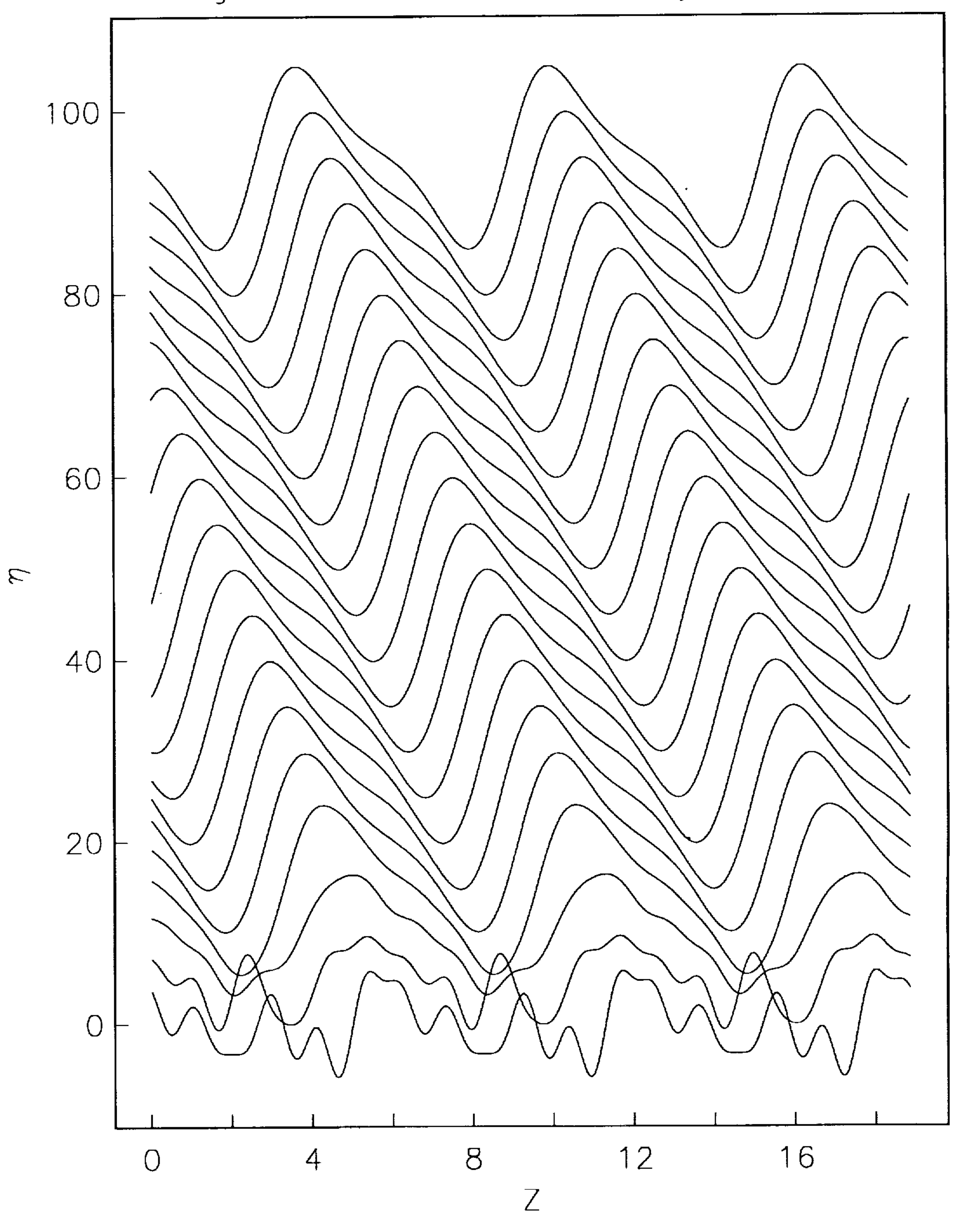
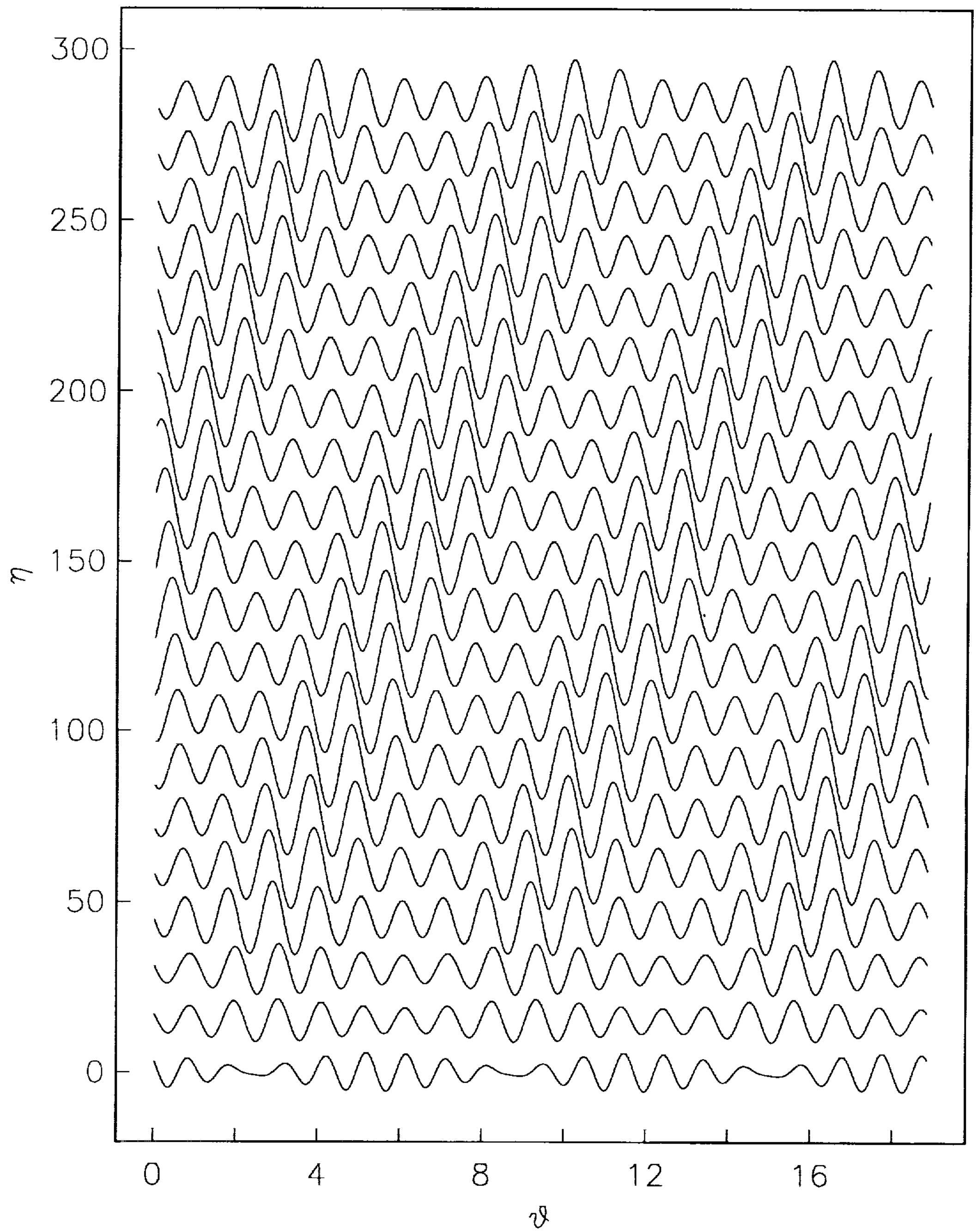


Figure 8 Solution η in ϑ



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Aidrian V. Coward
Philip Hall

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The interfacial stability of rotating core-annular flows is investigated. The linear and nonlinear effects are considered for the case when the annular region is very thin. Both asymptotic and numerical methods are used to solve the flow in the core and film regions which are coupled by a difference in viscosity and density. The long-time behaviour of the fluid-fluid interface is determined by deriving its nonlinear evolution in the form of a modified Kuramoto-Sivashinsky equation. We obtain a generalization of this equation to three dimensions. The flows considered are applicable to a wide array of physical problems where liquid films are used to lubricate higher or lower viscosity core fluids, for which a concentric arrangement is desired. Linearized solutions show that the effects of density and viscosity stratification are crucial to the stability of the interface. Rotation generally destabilizes non-axisymmetric disturbances to the interface, whereas the centripetal forces tend to stabilize flows in which the film contains the heavier fluid. Nonlinear affects allow finite amplitude helically travelling waves to exist when the fluids have different viscosities.

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